Reduction of optimization problem by combination of optimization algorithm and sensitivity analysis

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A special optimization technique based on combination of genetic algorithms and sensitivity analysis is proposed. The technique allows reducing the number of optimized parameters during the optimization process and, consequently, the time of optimization. The paper explains the principle and benefits of the algorithm and illustrates its utilization with a illustrative example. The algorithm is implemented in the framework OptiLab that represents a part of the application Agros2D developed by the authors.

***Index Terms*—genetic algorithms, sensitivity analysis, design optimization, finite element analysis, electromagnetic fields, microstrip**

I. Introduction

GENETIC ALGORITHMS (GAs) represent efficient tools for finding optimal solution from a large amount of possible solutions. For this reason, they are also frequently used for the optimization of various electrical devices. In a lot of cases, however, large and complex systems have to be solved that are characterized by a great number of the parameters to be optimized e.g. shape and topology optimizations. This also results in high demands on the solution of their mathematical models. When such a problem is formulated, it is extremely difficult to estimate which of these parameters have a high influence on the results and which not.

The paper presents an efficient combination of genetic algorithms and sensitivity analysis that allows quantifying the influence of partial parameters on the solution of the model describing the behaviour of the system. The implementation of the algorithm is intended mainly for the solution of tasks where the determination of every individual is represented by the solution of a partial differential equation or a set of such equations. The computation of the objective function is then highly time-demanding and, with respect to this fact, the time of realization of other parts of the algorithm is practically negligible. Utilization and benefits of this implementation is demonstrated on a simple example of a microstrip line, whose goal is to find geometrical dimensions providing the required characteristic impedance [4].

II. Proposed Algorithm

The described implementation of GA for single and multi-criteria optimization starts from the classic concept of GA, but it contains a number of techniques that are very practical for the optimization, where computation of every individual is time consuming. These techniques include, for example, application of discrete and continuous optimized parameters, definition of mutual relations among the parameters and mainly implementation of sensitivity analysis in the optimization cycles.

The last technique uses the calculated values of the objective functions F of already existing individuals and prior to creation of the next population the sensitivity analysis of partial parameters P_i is carried out (i denoting the index of the parameter). The sensitivity analysis represents a process of finding the dependence of the objective function $F(P_1, P_2, \ldots, P_n)$ on particular parameters (in the case of a multi-criteria optimization, the process is performed for every objective function). In order to quantify this dependence, the Pearson correlation is performed for every parameter in accordance with the relation

$$\rho_{F,P_i} = \frac{E\left[(F-\mu_F)(P_i-\mu_{P_i})\right]}{\sigma_F \sigma_{P_i}},$$

where E is expected value operator, μ is the mean value and σ is the standard deviation. Based on this correlation, the weight $w_{P_i} = |\rho_{F,P_i}|$ of every parameter is determined.

The correlation is always performed after the calculation of the given population and the weight of every coefficient is made more accurate. As far as the weight of any coefficient exceeds the threshold $w_{\min}$, the value of the parameter is fixed and does not change any longer.

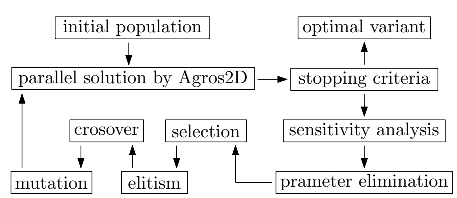

Fig. 1. Block diagram of optimization process by genetic algorithms

III. Illustrative Example

Functionality of the proposed approach was tested on a simple example whose result may be easily found in another way. It is a microstrip line representing a specific kind of electric line manufactured on a printed circuit. It consists of a conductor placed on a dielectric substrate. The ground electrode is located on its bottom side. The basic arrangement with the optimized parameters is depicted in Fig. 2.

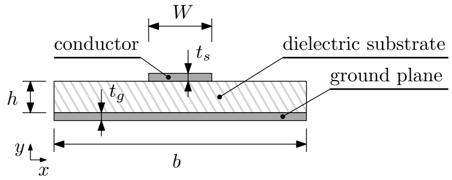

Fig. 2. Arrangement of optimized microstrip line

The most important parameter for its practical utilization is its characteristic impedance Z_0 given by the formula

$$Z_0 = \sqrt{\frac{L}{C}}\,, \tag{1}$$

where L denotes the inductance of the line and C is its capacitance. Both these quantities may be calculated from energies W_m and W_e of its magnetic and electric fields

$$L = \frac{2W_\mathrm{m}}{I^2}\,, \quad C = \frac{2W_\mathrm{e}}{U^2}\,,$$

where I is the current and U is the voltage of the conductor. The mathematical models of the magnetic field are given by the partial differential equation (PDE) in the form

$$\mathrm{curl}\ \left(\mu^{-1}\mathrm{curl}\ \boldsymbol{A}\right) = \boldsymbol{J}_\mathrm{ext}\,, \tag{2}$$

where μ is the permeability, $\boldsymbol{A}$ is the magnetic vector potential and $\boldsymbol{J}_\mathrm{ext}$ stands for the current density. The second PDE for the electric field reads

$$\mathrm{div}\ \left(\varepsilon \cdot \mathrm{grad}\ \varphi\right) = 0\,, \tag{3}$$

where ε is the permittivity and φ denotes the electric potential.

The aim of optimization is to find the required value of the characteristic impedance Z_0. The objective function is defined as the absolute value of the required and calculated impedance: $F = |Z_0 - Z|$. The optimization problem itself is defined in the standard form

$$\text{minimize} \quad F(P)\,, \quad P = (W, h, b, t_\mathrm{s}, t_\mathrm{g})\,,\ Z_0 = 75\,\Omega\,. \tag{4}$$

IV. Results

The optimization algorithm found the solution of the problem with value $F = 74.986\,\Omega$ ($Z_0 = 75\,\Omega$) after 10 populations, where every population contained 50 individuals. Figure 3 shows the evolution of weights w_i of particular parameters.

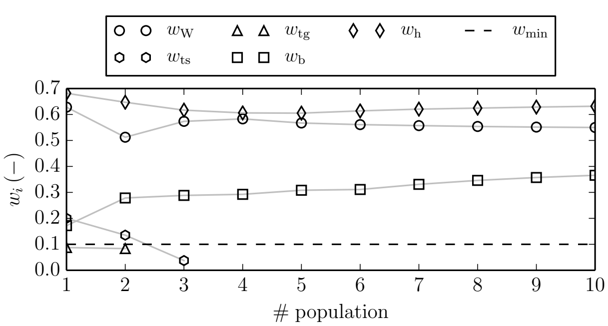

Fig. 3. Evolution of parameter weights during optimization process

It is obvious that the correlation criterion become more accurate during the process. The weights of parameters t_s and t_g representing the thicknesses of the conductor and grounding plate reached the threshold $w_\mathrm{min} = 0.1$, so that they were excluded from further computations.

Figure 3 shows that the parameters t_s and t_g have only minor influence on the result. This fact is also supported by the results depicted in Fig. 4 containing scatter plots for these parameter.

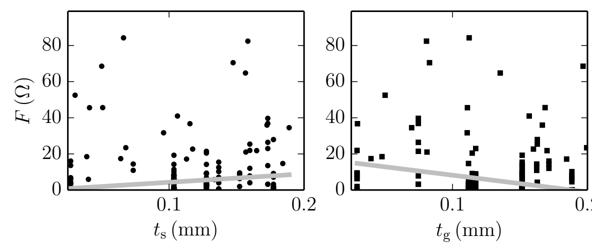

Fig. 4. Evolution of parameter weight during optimization process

V. Conclusion

The proposed adaptive algorithm allows reducing the number of the degrees of freedom of the problem. This leads to its simplification and a substantial reduction of computation time. The results of numerical experiments with the proposed algorithm shows its reduction on the order of tens of percent.

A very important task for near future will be the analysis of influence of parameter sampling on the results of sensitivity analysis connected with the optimization algorithm, because this algorithm samples mainly the subsets of search space around the extreme. The full paper will solve the shape optimization of an electromagnetic actuator described by a nonlinear mathematical model. More methods for sensitivity analysis will also be tested.

Acknowledgment

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